Professional development that targets understanding students' thinking through mathematical discourse

Summary

This strategy focuses on providing leaders with a set of tools to help them facilitate professional development sessions that target understanding students' thinking. The strategy helps leaders of professional development assist teachers in examining how interactions between the teacher, students, and curriculum influence the engagement with mathematical ideas and students' depth of understanding. A key role for a leader is to serve as a facilitator and model the role of mathematical discourse in understanding ways of reasoning. In using this strategy, leaders

- use and select substantial problems worthy of discussion
- facilitate rather than dominate discussions between teachers of their reasoning on substantial problems
- listen carefully to what teachers say and pose questions to deepen teachers' understanding of mathematical concepts, connections, or procedures
- engage teachers in a process of analyzing student discussions and written work for evidence of students' understanding
- provide access to the most current research available on teaching and learning so that teachers can examine and revise their assumptions about the mathematics they teach, what is important for students to know, how they teach it, how students learn mathematics
- create an environment where teachers explore problems, try out strategies in their own classrooms, and share what they learned, and
- value reflection as a powerful tool of growth and inquiry into a teachers' classroom practice.

The tools recommended here give leaders methods for engaging teachers in mathematical discussions and creating cultures in their classroom that values mathematical discourse. Each tool contains leader guides and glimpses into classrooms through student work, video, or teacher dialogues. This strategy uses high-quality resources (e.g., books, articles, video, online resources) to help leaders organize and lead professional development sessions that engage teachers in mathematical discussions and support inquiries into a teacher's practice.

Challenges Addressed

How do I engage students in mathematical discussions? Learning to communicate mathematically is at the heart of deepening understanding in mathematics for teachers and students, but teachers find it difficult to engage students in meaningful mathematical discussions.

The Strategy

Background

The National Council of Teachers of Mathematics published three standards documents: *Curriculum and Evaluation Standards for School Mathematics,* 1989; the *Professional Teaching Standards (PTS)*, 1991, and *Principles and Standards for School Mathematics PSSM*, 2000. These documents set standards for classroom discourse and describe the changing roles of teachers and students and tools for development in mathematics classrooms.

> The discourse of a classroom - the ways of representing, thinking, talking, agreeing and disagreeing - is central to what students learn about mathematics as a domain of human inquiry with characteristic ways of knowing. Discourse is both the way ideas are exchanged and what the ideas entail: Who talks? About what? In what ways? What do people write, what do they record and why? What questions are important? How do ideas change? Whose ideas and ways of thinking are valued? Who determines when to end a discussion? The discourse is shaped by the tasks in which students engage and the nature of the learning environment; it also influences them. (NCTM 1991)

The professional development standards in *PTS* include standards that support opportunities for teachers to

- experience good mathematics teaching,
- know mathematics and school mathematics,
- know students as learners of mathematics,
- know mathematical pedagogy, and
- continue to develop as a teacher of mathematics.

Currently, most teachers are very adept at sharing their own thinking, but may lack an ability to orchestrate a discussion that engages students' thinking, and thus reveals the multiple ways of reasoning about the same problem. Teachers who view teaching as telling will find it difficult to shift their views towards engaging students in mathematical discussions. These teachers control the mathematical discussions, provide answers to questions, and have the dominant voice in the classroom. If we are to impact the mathematical talk in classrooms, teachers need to increase their knowledge base on the research around the role of mathematical discourse and its relationship to understanding.

Achieving a change from the traditional well-understood didactic agreement between students and teacher in the classroom is not easy. The mathematical discourses in the classroom needs not only include interactions between students, the teacher, and text, but also between students—posing and answering their own questions, and suggesting inquiries for the classroom. The interactions in a classroom are quite complex and require new skills for teachers to engage all learners in a substantial mathematical discussion that promotes deeper understanding of the content. A teacher faces many conflicting ideas and challenges (e.g., time, curriculum, efficiency, and district expectations) that may pull them away from some discourse behaviors.

Implementing the Strategy

Professional development that engages teachers in exploring real questions, problems, and inquiries, over time, in ways that impact their perspectives, policy, and practice is viewed as the most promising. This type of professional development communicates a view of

teachers not only as classroom experts, but also as productive and responsible members of a broader professional community (Little, 1993).
There are a growing number of professional development programs that target using research-based knowledge to examine student's thinking. One of the primary goals in *Cognitively Guided Instruction (CGI)* is to share research-based knowledge about the development of students' mathematical thinking with teachers. Teachers who learn to focus on students' thinking by examining carefully grouped problems, and the strategies students use to solve problems are less likely to use direct instruction as a primary instructional strategy. Teachers who learn to listen to their students' thinking learn more about problems to pose, strategies to expect, and the connections between them. Focus on students' thinking is a powerful means for bringing pedagogy, mathematics, and students thinking together (Franke, Kazemi, 2001).

This strategy through the tools it suggests will help leaders guide teachers towards making effective mathematical discourse a reality in their classroom. The tools in this strategy share the following features: 1) asking questions that will deepen students understanding and reveal misconceptions, 2) listening to what students say, 3) analyzing videotapes that show students' thinking, 4) engaging teachers in solving problems and explaining their reasoning, 5) considering how students are likely to solve a problem, 6) reflecting individually or with peers about research-based ideas in relation to their own practice, and 7) considering instructional implications.

Implementing the strategy

Each tool includes videotapes that provide glimpses into another teacher's practice, along with a guide for organizing a series of workshops around different themes. Leaders should incorporate as many of the general principles from *Giving our professional development more impact* as feasible. Depending upon purposes for the professional development activities, and budget constraints, there are a variety of ways activities could be organized so they are grounded in specific classroom practices.

The professional development activities should be conducted over a period of time with breaks between sessions so that participants will have an opportunity to try some ideas and bring samples of student work to follow-up sessions. Leaders will need to identify the target audience (e.g., K-2, 3-5, secondary) and select tools that are best suited for the grade range and needs of the participants. A key feature of the strategy involves developing a community of learners that value reflection, risk-taking, and mathematical inquiry.

Choosing tasks

In order to engage students in mathematical discourse, it is best to use substantial mathematical tasks. Similarly, to engage teachers in mathematical discourse, it is best to use worthwhile mathematical tasks. These tasks are typically open-ended with different solutions possible, or open-middle tasks with one solution but multiple ways to approach the problem. The tasks should vary in length, type, mathematical content and processes. Typically, these types of tasks are non-routine and require explanations for how they reasoned about the mathematics. The tasks should have the potential to enhance teachers' mathematical knowledge and do not need to be intended for use in the teacher's classroom.

There are many published resources that contain mathematical tasks of this kind including, NCTM publications, some textbooks, and NSF curriculum project materials. The Eisenhower National Clearing House, http://www.enc.org, has many useful links and ideas that include lessons and activities, curriculum and professional development resources that teachers and leaders will find useful. The *Balanced Assessment: classroom packages* provide a collection

of high-quality performance tasks for different grade levels that can contribute towards development meaningful discussions around ways of reasoning. Teachers using traditional textbooks will find a range of tasks to supplement concepts that are being developed.

Traditional textbooks and instructional materials may have rich problems embedded in the problem sets. Typically, these textbooks have 40 to 60 exercises that range from developing skill competency to contextualized application problems. Leaders can help teachers make decisions about which problems to use to enhance and deepen students' understanding of a mathematical idea. The applications and problem solving exercises have the greatest potential for engaging students in mathematical talk and deepening their understanding.

Study Groups

Study groups may be organized by pairing grade-level teachers, groups of teachers at a grade band (e.g., K-2 and 3-5), or by mathematics course. Groups should decide which resources to use and establish a time-line for reading and reflecting. It is important for study groups to meet frequently, ideally, once a week, for a few months. Teachers may find it powerful to write their reflections in a journal and share things that worked (or didn't), and bring samples of student work for group discussion.

Leaders can facilitate the development of study groups an help organize the activities and discussions. The group should establish norms for sharing and providing feedback. The environment must be supportive and value risk-taking. It is important for a leader to communicate the value of refection as a powerful tool for school improvement and student achievement and seek the support of building administrators. Study groups typically meet outside of regular class time and may need an incentive to meet on a routine basis. The administrator needs to find creative ways to allocate time for groups to meet (e.g., staff meetings include time for study group discussion). The study groups could focus on specific discourse activities (e.g., examining student work, peer observations, sharing ideas from workshops) and share what they are learning. Additionally, videotaping and analyzing the mathematics lessons of to examine the range and type of questions teachers pose, and how students engage in mathematical talk in their classroom would be a valuable service.

Organize Summer Institutes, Workshops

Conducting a series of professional develop workshops requires an experienced facilitator and additional resources (e.g., stipends, facility, refreshments, copy costs). The facilitator will find ample support in the resources that are provided, but needs to be able to guide a discussion that involves risk-taking and engaging in doing mathematics which may awaken anxieties from teachers past experiences in doing mathematics.

Leaders may organize a series of activities that engage teachers in solving rich mathematics problems as a means of facilitating mathematical discourse. A series of workshops can be conducted periodically and organized around specific mathematical content (e.g., algebra: patterns & functions). Depending upon time and/or budget constraints, workshops can be conducted after school or teachers can be released from their assignment to participate in a half-day or full-day workshop. A minimum of 3 sessions should be organized to address different aspects of discourse (e.g., tools, environment, tasks). Participants should be expected to try out some ideas with their own students and share what they learned at the follow-up sessions.

A more intensive approach is to provide a summer institute that focuses on a broader range of mathematical content and processes. Leaders can use the tools and resources that are

identified in this strategy to map out an agenda for 1-3 weeks of institutes that focus on different aspects of mathematical discourse (e.g., environment, questioning, tasks, tools for discourse, curriculum). Initial activities could be devoted to establishing the adult learning community and setting a tone that values reflection as a powerful tool for growth. Participants in summer institutes should be encouraged to bring a colleague or teammate so they will have someone to talk to when they get back home and try out some of the ideas.

The *standards-based mathematics curricula* are based on a framework that develops and encourages mathematical discourse. The projects are designed to deepen students' understanding of mathematics. To learn more about the curriculum projects, related project research findings, and implementation studies visit these sites:

- NSF High School Curriculum Projects http://www.ithaca.edu/compass/
- NSF Middle School Curriculum Projects http://showmecenter.missouri.edu/showme/default.html
- NSF Elementary School Curriculum Projects http://www.comap.com/elementary/projects/arc/
- K-12 Mathematics Curriculum Center http://www2.edc.org/mcc/

Growing as a Professional Developer/Teacher Leader

Immersion in Educational Research

Professional development leaders need to stay familiar with the most current research that impacts teaching and learning mathematics for understanding. Leaders need to have an understanding of what the big ideas are for elementary, middle and high school students and the issues teachers may face as they try to implement research ideas into their own classrooms. *Adding it Up: Helping Children Learn Mathematics*, published by the National Research Council (2001) presents summaries of current research about school mathematics from pre-kindergarten to eighth grade; makes recommendations for teaching, teacher education, and curriculum for improving student learning, and provides guidance for educators, parents, and policy makers. Reading professional journals, attending conferences and networking are valuable activities for a professional developers continued growth. The research articles can be used with teachers to generate discussion around important mathematical content and processes that need to be used in the classroom. The National Council of Teachers of Mathematics organized three volumes, *Research Ideas for the Classroom* (1993) that put research within reach of teachers and leaders of early childhood, middle grades and high school mathematics classes. Each volume includes a synthesis of the research, implications for education, and connections to the classroom. Teachers are invited to engage in action research with their own students.

There are a growing number of teachers who are being thrust into leadership positions. When grounded in research, these teachers can be become powerful leaders of reform initiatives who can carry out professional development activities at a local level. Peer-to-peer collaborations are very productive when they focus on student work. Teacher leaders may be reluctant to appreciate the expertise they possess and will need time to develop confidence in their new roles as leaders. Networking with university educators, other schools and leaders will eliminate the isolation these emerging leaders may face.

Attend Summer Institutes, Workshops Designed for Leaders

There are many workshops and institutes that contribute to a teacher-leader or professional developer's growth in their ability to facilitate workshops and institutes for teachers. A team of 2-3 leaders would be ideal for when conducting extended sessions or intensive summer institutes. This team might include a new teacher-leader who can be mentored as

a potential leader of workshop sessions. Attending tool-specific workshops or institutes would give leaders deeper insights into a tool's development and how it can be implemented with varying audiences and circumstances.

- *Developing Mathematical Ideas* (DMI) Leadership Institutes are available in the summer from Center for the Development of Teaching, Educational Development Center, Inc. http://www2.edc.org/CDT/dmi/dmicur.html The curriculum includes
- *Cognitively Guided Instruction* (CGI) Institutes are designed for teachers of K-3 and professional development leaders. Participants learn to understand students' mathematical thinking. CGI helps teachers develop a framework for understanding children's intuitive mathematical thinking and also helps teachers learn to use knowledge of student's thinking to make instructional decisions. Annual CGI Institutes are presented by The Comprehensive Center, at Wisconsin Center for Education Research at the University of Wisconsin Madison. http://www.wcer.wisc.edu/ccvi/CGISpider/index.html
- *Assessing Math Concepts* (AMC) is a 5-day institute designed for teachers, teacher leaders, and those who provide professional development for K-3 teachers. AMC gives participants an in-depth view of how students learn mathematics and how to identify their instructional needs. *Teaching for Understanding* is a 5-day course for teachers of grades K-2 and 3-6 that provides methods of creating a learning environment that focuses on developing and assessing student understanding. *Thinking with Numbers* is a course where teachers learn to help develop students' computational fluency using a variety of models and strategies. There are numerous professional development offerings listed on the Math Perspectives website. http://www.mathperspectives.com/courses.html

Evaluative Evidence

Key features

Strengths

- Uses high-quality video, text, and electronic resources
- Provides resources that can be used for additional readings
- Supports development of mathematical discourse in classrooms based on research
- Provides examples of classrooms where teachers and students are exploring mathematical discourse

Likely Challenges

- This strategy depends upon a skilled facilitator. The facilitator must know how to engage adult learners in mathematical discussions and be sensitive to the struggles that may be revealed.
- Teachers need sustained professional development experiences. Districts and administrators need to support the ongoing professional development activities.

The Tools:

Talking Mathematics: Resources for Developing Professionals is a series of videotapes that can be used for a variety of inservice purposes. The materials include a leader's guide, along with videotapes that highlight different aspects of elementary students' and their teacher's engagement with activities that require them to explain thinking and reasoning. These resources were developed by TERC with support from the National Science Foundation. *http://www.heinemann.com* Cost: $445.50

Bridges to Classrooms Mathematics: Staff Developer's Guide, Mathematics for Elementary Teachers sessions are designed to strengthen teachers' mathematical understanding and confidence, as well as model pedagogy when using NSF supported curriculum materials. Each session includes a Staff Developer's Guide, masters for handouts, overhead transparencies, materials lists, and some may include a video. http://www.comap.com/elementary/projects/bridges/ Packages are available in various configurations (e.g., Comprehensive Package $287.00, Investigations Package $224.00, Everyday Mathematics Package $155.00). See website for details.

Kathy Richardson's Professional Development Videos

Each collection includes several sets of videos that focus on different aspects of a mathematical performance. All videos are available at http://www.didax.com

- *A Look at Children's Thinking*—assessment, includes 2 videos that focus on children's thinking and assessing their understanding, Costs $139.95
- *The Learning Environment*—what should the classroom look like when teaching math for understanding and concept development in the K-2 classroom Costs: $89.00
- Companion book: $29.95
- *Thinking with Numbers: Number Talks*—mental math 2 video set illustrates primary classrooms, Cost: $155.00
- *Making It Work in Your Classroom*—guide for supporting teaching for understanding, includes 2 videos. Cost: $155.00

http://www.mathperspectives.com/

Children's Mathematics: Cognitively Guided Instruction (CGI)

A research based professional development program that focuses on using research about children's mathematical thinking to guide instructional practices. The program is developed for grades K-3 on the premise that teachers who have a deeper knowledge of students' mathematical thinking will be more inclined to use that knowledge when making instructional decisions. The book includes 2 CD's that provides glimpses into the classroom where teachers are implementing the strategies based on research. http://www.heinemann.com Cost: $22.50

Developing Mathematical Ideas (DMI) Seminars currently there are five published seminars that are available, Part 1: *Building a System of Tens*, Part 2: *Making Meaning for Operations, Statistics: Working with Data, Examining Features of Shape, and Measuring Space in One, Two, and Three Dimensions,* available at http://www.pearsonlearning.com. The project is designed for teachers of K-5 and focus on the big ideas in the elementary curriculum and get to the core of how students understand mathematics. The materials include casebooks, a video, and facilitator guide that help leaders conduct sessions that provide insight into students' mathematical thinking. Facilitators' package for each seminar (Participant's Casebook, Facilitator's Guide and Videotape) costs: $85.95. Participant's Casebook cost: $34.95

Encouraging Mathematical Thinking: Discourse around a Rich Problem: provides an online "video-paper" that provides video, text, and tasks for teachers to use when learning to engage students in mathematical discussions. The site provides separate lesson plans of the "cylinder problem" for elementary through advanced high school mathematics classes. http://mathforum.org/brap/wrap/

Comments: The following articles provide additional insights into different aspects of mathematical discourses.

- ***Implementing Reform in the Mathematics Classroom: Creating Mathematical Discourse communities***, Edward Silver & Margaret Smith, share features of classrooms where students are expected to work on challenging problems and encouraged to communicate their mathematical thinking. http://www.enc.org/
- ***Questioning in the Mathematics Classroom***: The range and types of questions teachers pose have a direct impact on student learning and understanding. This article presents three categories of teacher questioning: factual, reasoning, and open, followed by a brief discussion of each type. http://www.enc.org/

References:

Franke M., Kazemi E. (2001). Learning to Teach Mathematics: Focus on Student Thinking. Theory into Practice, Volume 40, Number 2, Spring 2001

Little, J.W. (1993). Teachers' Professional Development in a Climate of Educational Reform. Educational Evaluation and Policy Analysis, Vol. 15, No. 2.,129-151

Professional Standards for Teaching Mathematics (1991). Professional Development Standards, 123-173