# Making Sense: Teaching and Learning Mathematics with Understanding

James Hiebert, Thomas P. Carpenter, Elizabeth Fennema, et. al. Heinemann Publications

### Summary

School systems have always regarded understanding as a critical component of the mathematics classroom, yet teachers continue to struggle with meaningful ways to teach for mathematical understanding. Teachers will likely agree that understanding involves more than procedural knowledge and that it includes the ability to reason with and make sense of what is learned, but they may lack ways to design a classroom so that understanding is the central to the students' learning experiences.

This book brings together ideas based on current research for designing classrooms that support students' mathematical understanding. It is based on the authors' individual work in

four separate research programs, each of which focused on instructional practices that support understanding. Out of their 5-year collaboration and discussions, key features emerged, some of which were identified as essential, and others as optional. The classroom stories provide insights into the different projects and classrooms that support students' mathematical understanding. *Making Sense* provides a framework for teachers to reflect on their own practice and consider or reconsider what it means to teach for understanding.



### **Purpose:**

This book provides guidance to think about designing classrooms where mathematical understanding is at the heart of instructional practices. It is intended to help

teachers of mathematics wrestle with the idea of "what is understanding" and how different interpretations and views about mathematics learning impact teaching.

### **Tool description:**

*Making Sense: Teaching and Learning Mathematics* with Understanding provides a framework that describes five dimensions that are critical to understanding:

- the tasks,
- the teacher's role,
- the social culture,
- tools, and
- equity.

The teacher's ability to integrate the dimensions will contribute to development of classrooms that support students' mathematical understanding and sense-making abilities. This knowledge will extend the boundaries of the classroom and be useful and accessible to students in their lives. The book defines mathematical understanding and helps frame the issues related to teaching and learning for understanding. The essential features of each of dimension is fully described and supplemented with illustrative examples to help teachers understand how to integrate the dimensions into their own classrooms.

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### Background

The authors were members of a working group of the National Center for Research in Mathematical Sciences Education at the University of Wisconsin-Madison. The purpose of the group was to consider the teaching and learning of whole number arithmetic in elementary school. This book grew out of a discussion that took place over a five year period of time. The authors were each involved in projects that focused on essential classroom features that facilitate mathematical understanding. The author's individual projects had differences and similarities that caused them to think more deeply about the critical features of classrooms that support students' mathematical understanding. The projects include: Cognitively Guided Instruction (Carpenter & Fennema), Conceptually Based Instruction (Hiebert & Wearne), The Problem Centered Learning approach (Human, Murray, Olivier), and Supporting Ten-Structured Thinking (Fuson). James Hiebert is professor of educational development in the College of Education, University of Delaware. Thomas P. Carpenter and Elizabeth Fennema are professors of curriculum and instruction at the University of Wisconsin- Madison. Karen Fuson is professor in the School of Education and Social Policy at Northwestern University. Piet Human is professor of mathematics education, and Hanlie Murray and Alwyn Oliver are senior researchers of mathematics education at the University of Stellenbosch, South Africa. Diana Wearne is associate professor of educational development at the University of Delaware.

### **Design principles**

The book is divided into four sections: 1) an introductory chapter that provides an overview and raises questions for the reader to explore and reflect upon throughout the book, 2) five chapters that describe each critical dimension for classrooms that are designed for mathematical understanding, 3) classroom episodes or stories that share how the dimensions might look when in action, and 4) the final section summarizes the way the dimensions work together in the classroom. The classroom episodes are based on the four different research projects and feature primary classrooms from diverse cultures.

# The materials

#### **Contents:**

- 1. Introducing the Critical Features of Classrooms
- 2. The Nature of Classroom Tasks
- 3. The Role of the Teacher
- 4. The Social Culture of the Classroom
- 5. Mathematical Tools as Learning Supports
- 6. Equity and Accessibility
- 7. A Day in the Life of One Cognitively Guided Instruction Classroom
- 8. A Day in the Life of a Conceptually Based Instruction Classroom
- 9. Student Talk in a Problem-Centered Classroom
- **10.** Snapshots Across Two Years in the Life of an Urban Latino Classroom
- **11.** Revisiting the Critical Features of Classrooms
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### Using the tool

Readers are invited to read the chapters and reflect on their own practices.

### **Evaluative evidence**

### Availability

See Heinemann Publications, http://www.heinemann.com

### Strengths

- based on research from four separate research projects
- features classroom stories from different projects that illustrate the critical features of a classroom that supports teaching and learning for understanding
- frames issues related to teaching for understanding
- provides a framework for designing classrooms that support teaching and learning for mathematical understanding

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# Likely challenges

- tensions between developing skills and teaching for understanding
- lack of instructional materials and tasks that engage students' thinking