

Developing Mathematical Ideas (DMI)

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Summary

Developing Mathematical Ideas (DMI) is a series of professional development seminars designed to help teachers think through the big ideas of K-7 mathematics and examine how these ideas develop in children. The materials include sets of classroom episodes (cases) that illustrate students' thinking as described by their teachers. The seminars offer teachers opportunities to explore mathematical ideas; closely examine the work of their own students; analyze videotaped interviews of students and learn to interview their own students; analyze mathematics lessons; read and discuss relevant research; and engage in a writing process that describes their own classroom episodes.

DMI seminars are designed to bring together teachers from kindergarten through middle grades to learn:

- more mathematics content to deepen their own understandings and make connections, thereby, enhancing their ability to help students
- to recognize key mathematical ideas their students are grappling with
- to understand and support the power, flexibility, and complexity of students' thinking
- to ask questions that will explore and deepen students' thinking
- how core mathematical ideas develop across the grades
- how to *continue* learning about children's thinking and mathematics

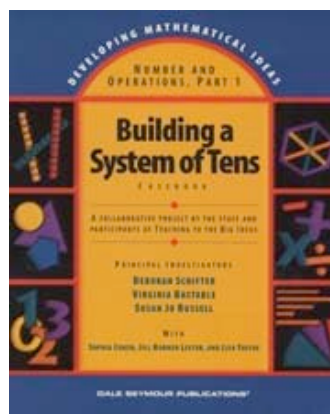
DMI has served as core curriculum for programs designed for teacher-leaders, administrators, parents, and for pre-service teachers. The materials have been used in both summer institutes and school-year settings.

Purpose

This tool provides leaders with professional development resources, both text and video images that can be used as a curriculum to engage teachers to explore more deeply the mathematics of the elementary and middle grades and to inquire into their mathematics teaching practices and its impact on students' learning.

Tool description

The *Developing Mathematical Ideas* facilitator's guide suggests ways to organize a series of seminars to engage teachers in "doing" mathematics, reading and discussing classroom episodes (cases), discussing and reflecting on teacher's practice, and reflecting on children's thinking through print and videotaped images of classrooms. The resources also include tasks that can be used to deepen teacher's mathematical knowledge, engage them in rich discussions about the mathematics, and help teachers learn to support and deepen students' thinking. Each Facilitator's Package includes a videotape, facilitator guide, and a casebook.



Packages and Casebooks

- **Number and Operations (Part 1): Building a System of Tens**
Participants explore the base-ten structure of the number system, consider how that structure is used in multidigit computational procedures, and examine how basic concepts of whole numbers reappear when working with decimals.
- **Number and Operations (Part 2): Making Meaning for Operations**
Participants examine the actions and situations modeled by the four basic operations. The seminar begins with a view of young children's counting strategies as they encounter word problems, moves to an examination of the four basic operations on whole numbers, and studies the operations in the context of rational numbers.
- **Working with Data**
Participants work with the collection, representation, description, and interpretation of data. They learn what various graphs and statistical measures show about features of the data, study how to summarize data when comparing groups, and consider whether the data provide insight into the questions that led to data collection.
- **Examining Features of Shape**
Participants examine aspects of 2D and 3D shapes, develop geometric vocabulary, and explore both definitions and properties of geometric objects. The seminar includes a study of angle, similarity, congruence, and the relationships between 3D objects and their 2D representations.
- **Measuring Space in One, Two and Three Dimensions**
Participants examine different attributes of size, develop facility in composing and decomposing shapes, and apply these skills to make sense of formulas for area and volume. They also explore conceptual issues of length, area, and volume, as well as their complex inter-relationships.

Background

Developing Mathematical Ideas is a professional development curriculum that grew out of the Teaching to the Big Ideas project (TBI). TBI was a 4-year professional development project, funded by the National Science Foundation, designed to develop elementary teachers' mathematical understanding and its impact on instruction. The project was collaboration among EDC, SummerMath for Teachers at Mount Holyoke College, TERC, and 36 teachers. The project aimed to address the interaction of teachers with curriculum reform and to study—project staff and teachers, together—the big mathematical ideas across grade levels in elementary mathematics. As both a mechanism for their own professional development and as data to explore how mathematical ideas arise in different classroom settings, participating teachers wrote classroom episodes that captured their students' mathematical thinking. The teachers' writings were initially intended for project use only, but it wasn't long before the staff realized the value of the cases for professional development experiences outside of the project. The use of the TBI cases became central to the development of the professional development curriculum, which was later named Developing Mathematical Ideas.

Early in the project, the TBI teachers made a commitment to organize and conduct some type of professional development activity at their home schools. They began to use the DMI materials once they became available. Of the 36 teachers who began the project, 30 remained with the project for the entire 4-year period of the grant.

Design principles

The seminars incorporate the following types of activities: Teachers

- working on mathematics and reflecting on their own mathematical thinking,
- reading and discussing the print cases,
- viewing and discussing the video cases,
- exploring innovative curricular materials,
- reading and discussing highlights of relevant research,
- writing cases that examine their own students' mathematical thinking, and
- reflecting on the major ideas in the mathematics curriculum and how they develop across the grades.

Using the tool

Materials for Participants: The Casebooks

Each seminar is built around a casebook containing 25 to 30 cases that focus on a particular mathematical theme (e.g., Measuring Space in One, Two, and Three dimensions) and its development from kindergarten through grade seven. Casebooks are organized around chapters that focus on sets of cases around a sub-theme (e.g., Measuring length: What is a unit and how is it used?). The casebook concludes with sections on related research so participants can frame their thinking within the context of present research that is made accessible to teachers.

Materials for Facilitators: The Facilitator's Guide and Video Cases

The DMI Facilitator's Guides include detailed agendas for each session. Other features are designed to help facilitators: lead case discussions and mathematics activities, plan and organize seminar sessions, understand the major ideas to be explored in each session, and think through issues related to teacher change.

The DMI Video Cases show students in diverse classroom settings with children and teachers representing different races and ethnicities. The video cases complement the casebooks and offer opportunities to listen to student voices and provide rich images of classrooms that are organized around student thinking.

DMI Leadership Institutes

The **DMI Leadership Institute Program** is designed for teams of staff developers, teacher-educators, teacher-leaders and others who support teachers' professional development in mathematics K-6. Institute participants make inquiries into the goals of professional development for elementary and middle school mathematics, reflect on the kinds of support structures and activities needed, and become familiar with DMI as a tool to improve mathematics teaching and learning at their sites.

Two levels of DMI Leadership Institutes will be offered in the summer of 2005. DMI Leadership Institute 1, based on Building a System of Tens and Making Meaning for Operations, is appropriate for educators who are new to the DMI materials. DMI Leadership Institute 2 builds on the work of DMI Leadership Institute 1 and will also include opportunities for participants to become familiar with the geometry,

measurement, and data units. Both levels of the two-week programs will take place July 17 through July 29 at Mount Holyoke College in Massachusetts. The program fee is \$1900 and includes room, board, and four graduate credits. The DMI Leadership Program is a collaboration between Educational Development Center and SummerMath for Teachers at Mount Holyoke College. It is partially funded by the ExxonMobil Fund.

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Mount Holyoke College
South Hadley, MA 01075-1441
Call (413) 538 - 2063
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E-Mail smt-dmiinfo@mtholyoke.edu

Please indicate that you are interested in the DMI Leadership Institutes.

Evaluative evidence:

The Teachers' Professional Development and the Elementary Mathematics Classroom: Bringing Understandings to Light, Sophia Cohen, published by Lawrence Erlbaum Associates, Inc., http://www2.edc.org/CDT/dmi/dmi_BUTL.html, provides the evidence through stories of how teacher's knowledge and learning changed as a result of their professional development experiences in the DMI project. Some of the findings include:

- Teachers came to believe they and their students have ideas about the subject of study and they can make sense of mathematics.
- Students' ideas became central to the classroom discussions and activities.
- Teachers' knowledge of mathematics they taught deepened and their teaching practices shifted to build on the students' understandings as they constructed new ideas.

Availability

Copies of the Facilitator's Packages and Casebooks may be obtained from Pearson Learning, <http://www.pearsonlearning.com>

Strengths

- Supports development of a leader's ability to ask questions that probe and challenges teachers' beliefs about what it means to know and to do mathematics
- Supports development of teachers' ability to ask questions to probe and deepen students' thinking
- Includes "Maxine's Journal" which was designed to convey what a DMI seminar might look like, the types of discussions that can take place, the type of lessons the participants might gain from the experience, and how it might feel to facilitate sessions

- Includes interesting problems to engage teachers in “doing” mathematics along with some discussion about potential use of the problems with students
- Uses high-quality videotapes
- Provides context for using videotapes, discussion questions, and classroom episodes presented through cases
- Provides classroom-based examples of students’ thinking written by teachers
- Encourages teachers to write their own cases
- Includes essays on recent research findings
- Supports development of a reflective community of learners that continue to learn more mathematics and about students’ thinking

Likely challenges

- Requires a commitment from teachers and districts for sustained, long-term professional growth.
- Requires the cultivation of a learning community that values reflection and risk taking.
- Requires facilitators knowledgeable about mathematics and pedagogy—or willing to learn this content more deeply.
- Requires facilitators who are attentive to the process of teacher change. In particular, many teachers need support as they encounter ideas that are new; others need to be challenged to delve more deeply into mathematics that is familiar.

For more information

To order materials from Dale Seymour Publishers, call (800) 321-3106.

DMI Leadership Institutes are held for two weeks in the summer at Mount Holyoke College, located in South Hadley, Massachusetts. These institutes are designed for teacher educators, teacher leaders, and others who support teachers' professional development. Visit the DMI web site at, <http://www.edc.org/LTT/CDT/DMIcur.html>.

To request additional information and an application form, write:

SummerMath for Teachers
 Mount Holyoke College
 South Hadley, MA 01075-1441
 Call (413) 538 - 2063
 Fax (413) 538 - 2002
 E-Mail smt-dmiinfo@mtholyoke.edu

Please indicate that you are interested in the DMI Leadership Institutes.

Some samples from the materials

The Casebook for each module includes 20 to 30 cases, written by teachers, organized in chapters that focus on a particular set of mathematical ideas that develop from kindergarten through grade seven. The facilitator’s guide discusses how to use the cases, organize discussions, and use related video clips and mathematics activities through detailed agendas. “Maxine’s Journal” was created to share a sense what a DMI seminar might look like, the types of discussions that can take place, the type of lessons the participants might gain from the experience, and how it might feel to facilitate sessions.

Below you will find the table of contents from the Casebook and Facilitator’s Guide for *Building a System of Tens, Part 1*.

Building a System of Tens, Number and Operations, Part 1: Casebook

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Building a System of Tens, Number and Operations, Part 1: Facilitator's Guide

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Complementary tools:

Talking Mathematics: Supporting Children's Voices

Writing to Learn: Strategies that Work, K-12

*Principles and Standards for School Professional Teaching Standards Talking
Standards-based classroom teaching materials*

Balanced Assessment for the Mathematics Curriculum: assessment packages

Classroom Discussions: Using Math Talk to Help Students Learn