# Turn Test Prep into Learning

MARS: Mathematics Assessment Resource Service, Michigan State University

**Summary**
This tool contains the following materials:

- an introduction that describes the purposes, components, and a routine for using this tool in your classroom;
- collections of 2-4 related multiple-choice tasks, each collection built around a common mathematical strand and problem context;
- samples of students' written work on task collections;
- a sample lesson plan and teacher commentary on task use in classrooms.

An exemplar of an upper elementary task collection from the Algebra strand is provided at the end of this document.

### Purpose

To provide teachers with instructional materials (and guidance for their use) to enhance students' learning while preparing students for high-stakes multiple choice assessments.

### Tool Description

The *Turn Test Prep into Learning* tool is designed to support teachers who want to help prepare students for high-stakes assessment on typical multiple choice tests in ways that are educative, that reveal misconceptions and faulty reasoning, and that promote further learning. Many teachers feel considerable pressure to devote weeks of classroom time to practice on multiple-choice items. The typical routine is to have students answer questions that are posed in a multiple-choice format and then check for the correct answer. This might be followed by asking students who answered correctly to show what they did. This procedure ignores the misconceptions that students might have about key aspects of an item or the faulty reasoning they may have used to answer the question, even if they chose the correct response.

### Tool Materials

This tool provides materials that teachers can use to prepare students for these kinds of tests while also enhancing student learning. The tool is composed of the following materials:

***Teacher's Guide:*** a routine for classroom use

***Task Sets:*** collections of related multiple-choice tasks, organized into sets around a common mathematical idea. Each task set contains the following materials:

- Tasks: 2-4 related items that target a typical benchmark, and that vary in presentation, context, distracters, or question asked
- Student work: samples of students' written work that highlight students' solution strategies
- From the Classroom: teacher commentary on tasks in the set that highlight issues of misconceptions and faulty reasoning. Some tasks also include a short vignette of classroom discussion.

### Evaluative Evidence

This tool is under development. Feedback from users is welcome.

**Strengths of this tool**

- features multiple-choice items released from various states;
- collections of tasks and variants include different content strands and processes;
- includes suggestions for classroom use and teacher commentary;
- task sets vary in levels of difficulty and can serve multiple purposes;
- provides samples of student work.

**Likely Challenges**

- requires teachers to handle classroom discussions less directively;
- takes more time to engage students in a discussion.

**Availability**

The tool is under development through trials in invited representative classrooms. For more information, contact *wilcoxs@msu.edu*.

**Exemplars**

The following sample features tasks from the Algebra strand that were used in different upper elementary classrooms, along with teacher commentaries.

# Turn Test Prep Into Learning
## *Teacher's Guide*

### Preparation

1. Select a task set from a content strand that targets a benchmark you want to work on with your students. Identify which other aspects of problem solving you wish to work on. These may include:
    a. **Stressing the importance of reading problems carefully**. In this case you may wish to give two versions of a task that are similarly worded but which lead to differing answers.
    b. **Exploring a range of alternative approaches to solving a problem**. It is important to have discussions that broaden students' range of strategies for tackling problems.
    c. **Showing the power of different representations**. This might mean that you select a problem in which students have to choose their own diagrams, tables, or graphs.
    d. **Alerting students to common misconceptions and mistakes**. These are often contained within the distracters.
2. Before giving the task set to your students, answer the questions yourself using the knowledge they are likely to bring to the task. Anticipate different ways they might reason and different strategies they might use to solve the problem.
3. Look at the samples of student work that accompany the task set. They may provide additional solution paths that you had not thought about.
4. Read "From the Classroom." This is an example of how one teacher led a whole class discussion. It illustrates how talking about and justifying ways of solving a problem reveal insights into students' thinking that their written work may obscure.
5. Decide whether you want to use some or all of the variations of the item.
6. Think about how you want to facilitate a discussion after your students have answered the questions.
7. Make copies of the task(s) you choose – one per student. (You can put two tasks on a single page if you choose to use more than one). Also make an overhead transparency of each item.

### Lesson Plan

1. **Introduction** (5 minutes)

   Distribute the task(s) to your class. Have a student read the task(s) aloud. Instruct students to ***show their work*** as well as to circle their answer.
2. **Individual Work** (5 minutes)

   Give students about 5 minutes to solve an item. Observe them as they solve the problem, looking for a variety of correct strategies as well as evidence of misconceptions, faulty reasoning and procedural errors.

3. **Pair / Small Group Work** (10 minutes)

   Have students first discuss their work in groups of 2-4. Each student tells what they got for the answer and how they solved the problem. Together they decide whether each approach is reasonable and agree on a correct solution.

4. **Discussion** (20-30 minutes)

   Have a whole class discussion about the various strategies they used to solve the problem. This discussion can help uncover whether some students chose a correct answer using an incorrect method. Even those that have a correct method may find it hard to explain it clearly. This discussion could be organized as follows:

   a. Take a poll to see how many students circled each of the distracters. Start with those distracters that were less popular. Discuss each distracter in turn.
      - How might a student have been reasoning to choose this answer?
      - What could they be thinking?
      - What mistake might they have made?

   b. Ask a few students to describe how they obtained the correct answer. (You don't need to reveal which one is correct at this stage!) You may want to have some students go to the overhead to explain what they did to the rest of the class.

   c. As an extension, ask the class to suggest other good distracters. This will give some an opportunity to describe their own errors in a non-threatening way.

5. **Optional Assignment** (5 minutes)

   a. Choose another problem from the task set. Have students take it home to solve. In their solution, they should show their work and explain how they solved the problem.

   b. Give students another problem with answer choices not listed. Have students generate all four multiple-choice responses – the correct answer and three reasonable distracters.

# **Class Picture** Task Set

Grade: **Upper Elementary**
Strand: **Algebra**

The items in this collection involve recognizing and extending a pattern and solving a problem using a pattern. Class Picture 1, 2, and 3 differ primarily in the presentation of the task: Class Picture 1 presents a drawing, Class Picture 2 provides only a written description of the problem, Class Picture 3 presents a partially completed table. Class Picture 4 has the same context as the others but poses a different and more challenging question. Each of the variants is typical of what students might encounter on a high-stakes test.

This collection of tasks was adapted from Florida State FCAT Sample Test Materials, http://www.firn.edu/doe/sas/fcat/fcatit01.htm

Following the tasks are vignettes of how a teacher used these items in two different classes. You may find it helpful to see how another teacher used these problems to engage children in a discussion about their reasoning and what the task has the potential to reveal about students' thinking.

## Class Picture 1

A fourth grade class at Elm Street Elementary School stood in rows to have their picture taken. The photographer told 2 people to stand in the first row, 4 people to stand in the second row, and 6 people to stand in the 3rd row.

If the pattern continued, how many people did the photographer ask to stand in the 5th row?

a) 8

b) 10

c) 12

d) 30

## Class Picture 2

A fourth grade class at Elm Street Elementary School stood in rows to have their picture taken. The photographer told 2 people to stand in the first row, 4 people to stand in the second row, and 6 people to stand in the 3rd row.

If the pattern continued, how many people did the photographer ask to stand in the 5th row?

a) 7

b) 8

c) 10

d) 12

## Class Picture 3

A fourth grade class at Elm Street Elementary School stood in rows to have their picture taken. The photographer told 2 people to stand in the first row, 4 people to stand in the second row, and 6 people to stand in the 3rd row.

| **Row** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Number of people** | 2 | 4 | 6 | | | |

If the pattern continued, how many people did the photographer ask to stand in the 6th row?

a) 7

b) 8

c) 10

d) 12

## Class Picture 4

A fourth grade class at Elm Street Elementary School stood in rows to have their picture taken. There are 30 people in the class. The photographer asked students to stand in rows so that each row increased by two people. There were two people in the first row.

How many people were in the last row?

a) 8

b) 10

c) 12

d) 30

## From a 5th Grade Classroom — Class Picture 2

*I examined each version of Class Picture and decided Class Picture 1 and 3 would be too easy for 5th graders. I chose Class Picture 2 and 4 because they allowed students to choose their own representations and I thought we could have a richer discussion about their ways of reasoning.*

*I handed out copies of Class Picture 2 and 4 and asked students to read the problems and show how they solved them on paper. Initially, several students thought both versions were the same. I prompted them to read each problem carefully. When students had sufficient time to work both problems, about 15 minutes, I brought the class back together to discuss their ways of reasoning about Class Picture 2. I asked how many got each of the answers and made a tally at the overhead.*

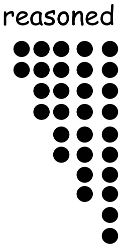
| | | |
|---|---|---|
| a) | 7 | none |
| b) | 8 | //// |
| c) | 10 | ~~////~~ /// |
| d) | 12 | // |

*I began the discussion by asking about the three answers that were not correct.*

**Teacher:** No one chose seven as a response, why isn't that a good choice?
**Branden:** The pattern's going by two's, seven isn't a multiple of two.
**Teacher:** We had four students who chose eight. Right now, I don't want the students who chose eight to tell us how they were thinking. I'd like for others to consider how these four students might have been reasoning.
**Alec:** Eight would probably be in the fourth row. Ten would be in the fifth row.
**Nia:** Oops! That's exactly what I did! I counted by two's until the fourth row. I didn't read the part about the fifth row.
**Elena:** That's what I did too!
**Teacher:** How might students have reasoned to choose twelve as the number of students in the fifth row?
**T.C.:** They probably kept counting by two's and lost track of where they were. Twelve would be in the sixth row.

*Then, I took up the correct response.*

**Teacher:** Many of you thought ten was the correct solution. Does Alec's reasoning seem plausible?
**Nia:** What's plausible?
**Teacher:** Is his reasoning a possible way these students might have been thinking? *(several students agreed)*
**Teacher:** Who chose ten, but reasoned about it a different way?
**Mark:** I drew a picture.



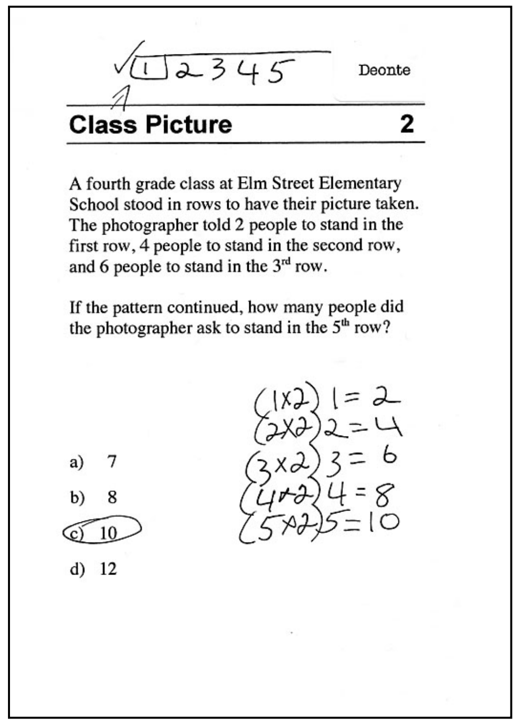
**Teacher:** How many of you drew some type of model? (*Four students raised their hands*)

*I then asked if anyone had reasoned about it differently and Deonte (whose written work is shown) offered to show how he thought about it.*

**Deonte:** I saw two, four, six and I started to write:

1 x 2 = 2
2 x 2 = 4
3 x 2 = 6
4 x 2 = 8
5 x 2 = 10

**Teacher:** What do the numbers stand for?
**Deonte:** One, two, three, four, and five are the rows.
**Teacher:** Why did you multiply by two?
**Deonte:** Because, that's how I saw it.


1 2 3 4 5 Deonte

**Class Picture 2**

A fourth grade class at Elm Street Elementary School stood in rows to have their picture taken. The photographer told 2 people to stand in the first row, 4 people to stand in the second row, and 6 people to stand in the 3rd row.

If the pattern continued, how many people did the photographer ask to stand in the 5th row?

(1x2) 1 = 2
(2x2) 2 = 4
(3x2) 3 = 6
(4x2) 4 = 8
(5x2) 5 = 10

a) 7
b) 8
c) 10
d) 12

*I asked him if T.C.'s reasoning about getting 12 for the sixth row fit his way of thinking and he responded "Yeah, because six times two equals twelve." Deonte immediately recognized a multiplicative relationship between the row and the number of people in each row, even though he wasn't able to clearly articulate how he saw it.*

## From a 5th Grade Classroom — Class Picture 4

*We then turned our attention to Class Picture 4. I made another tally at the overhead of how students answered this item.*

a) 8
b) 10 卌
c) 12 ///
d) 30 //
no answer ////

*I began the discussion by asking how Class Picture 4 was different from Class Picture 2.*

**Branden:** This one tells you there are thirty students in the class.
**Sam:** It's asking you how many people are in the last row. The other one asked for the fifth row.
**Teacher:** I want you to think about how a student might have been reasoning to get thirty as a solution. Again, I don't want the students who chose thirty to tell us how they were thinking. I'd like for others to consider how they might have been reasoning.
**Gemma:** Thirty doesn't make sense. It's impossible to have thirty people in the last row and two people in the first row. There are thirty people all together!

*Deonte, who was one of the students who got 30 for his answer, spoke up.*

**Deonte:** I did the same thing as the first problem. 1 = 2 (*writes on overhead*)
2 = 4
3 = 6
**Teacher:** Let's agree to not use an equal sign. These two numbers are not equal. What you are saying is one goes to two, two goes to four, so let's use an arrow. *(Deonte continued)*

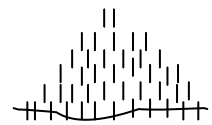
$4 \rightarrow 8$
$5 \rightarrow 10$
$6 \rightarrow 12$
.
.
$15 \rightarrow 30$

**Deonte:** The rows are one, two, three, four. . . and the people increase by two *(wrote "increase by 2" next to the column)*, so thirty people.
**Teacher:** Does this make sense? What does thirty stand for?
**Jada:** I got the same thing that Deonte did. (*she joins Deonte at the overhead*). See fifteen and fifteen makes thirty. (*Deonte writes 30 ÷ 15 = 2, and nods in agreement*)
**Gemma:** That still doesn't make sense. If two people are in the first row, that only leaves twenty-eight people. There can't be thirty people in the last row.

**Teacher:** Deonte, Jada, and others, what do you think? (*unconvinced, they sat down*) I need you to listen to each other's thinking. It may become necessary for you to let go of an idea if it doesn't make sense.

**Teacher:** Alec, would you draw us a model of how you thought about the problem?

**Alec:** I used marks for people. (*Counting marks as he drew them*) one, two, three, four, five, . . . thirty, thirty-one, thirty-two . . .



**Teacher:** How many people do you need?

**Branden:** He should stop at thirty. (*He crossed off the last row.*)

**Teacher:** How many people are in the last row?

**Branden:** Ten people.

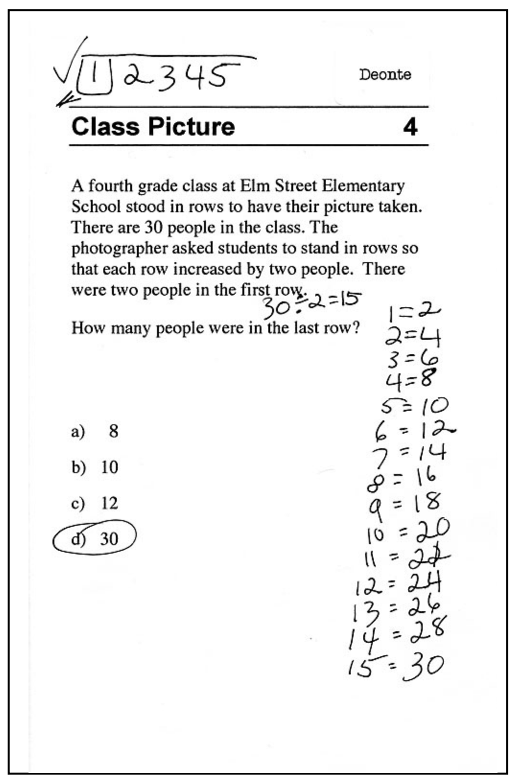
**Teacher:** I think Deonte and Jada were answering a different question. Does anyone know what question they answered?

**Deonte:** There were thirty children in the last row and thirty-two would be in the sixteenth row.

**Teacher:** That's the question that you and Jada answered. You didn't answer the question about the number of children in the last row if there were a total of 30 children all together.

*At this point, our discussion had gone on for nearly an hour. This last part, where Deonte was so insistent about his reasoning, left us all exhausted. I decided to leave it for the moment.*

*I think Deonte's multiplicative approach is a promising way to help students look for relationships between variables. I wanted to push their thinking towards generating a rule that would work for this situation (row number x 2 = number of people in the row). But we can return to this problem another time. I didn't anticipate that the analysis of the two tasks would take an entire class period. However, I'm convinced that mathematical reasoning, justification, and argumentation can only take place when students are given an opportunity to consider approaches other than their own. It is in presenting ideas to an audience that students attempt to justify their solutions and engage in a sense-making conversation.*



√ 1 2 3 4 5

Deonte

**Class Picture** 4

A fourth grade class at Elm Street Elementary School stood in rows to have their picture taken. There are 30 people in the class. The photographer asked students to stand in rows so that each row increased by two people. There were two people in the first row.

30÷2=15

How many people were in the last row?

1=2
2=4
3=6
4=8
5=10
6=12
7=14
8=16
9=18
10=20
11=22
12=24
13=26
14=28
15=30

a) 8

b) 10

c) 12

d) 30

## From a 4th Grade Classroom — Class Picture 3

*I chose Class Picture 3 for my fourth grade students because exploring patterns with tables is an expectation at this grade in my district. After the students had worked for about 10 minutes on the task, I asked if someone would like to explain how they solved the problem.*

*Akira went to the overhead and began to write on a transparency of the task.*

**Akira:** It is twelve (*as she circles answer choice d on the transparency*). I got two plus four plus six equals twelve (*as she writes her number sentence on the transparency next to the answer 12 she just circled*).

**Teacher:** How did you think about it?

**Akira:** mmmm…because two plus four is six and plus six is twelve

**Teacher:** Is this how you did it on your paper?

**Akira:** …mmmmm…no…

**Teacher:** How did you do it on your paper?

**Akira:** I said…look, eight, ten, twelve (*as she writes 8, 10, 12 just below the question and above her number sentence. Then she writes these three numbers in the empty spaces in the table – 8 below 4, 10 below 5, and 12 below 6. She then draws arrows to show how the numbers she wrote in the table match the numbers she wrote below*).

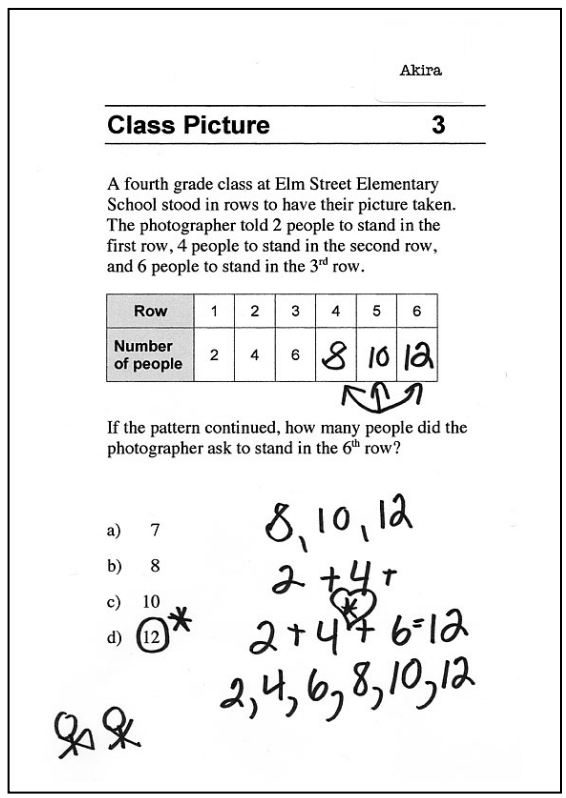
**Teacher:** What are those?

**Akira:** That's how I found twelve…goes up by two (*and draws two stick people and next to them the numbers 2, 4, 6, 8, 10, 12*).

**Teacher:** But why did you say then two plus four plus six?

**Akira:** Because it is twelve and that's why the result is twelve…

*Akira may have thought that her way of using the pattern she saw, "goes up by two," to find the answer was not an acceptable way to solve the problem – or at least to justify her answer. We have done work with problems where students need to write a number sentence to show how they applied a rule. And while that did not apply to this situation, Akira may have thought she needed to write some kind of a number sentence to show a result of 12.*


Akira

**Class Picture 3**

A fourth grade class at Elm Street Elementary School stood in rows to have their picture taken. The photographer told 2 people to stand in the first row, 4 people to stand in the second row, and 6 people to stand in the 3rd row.

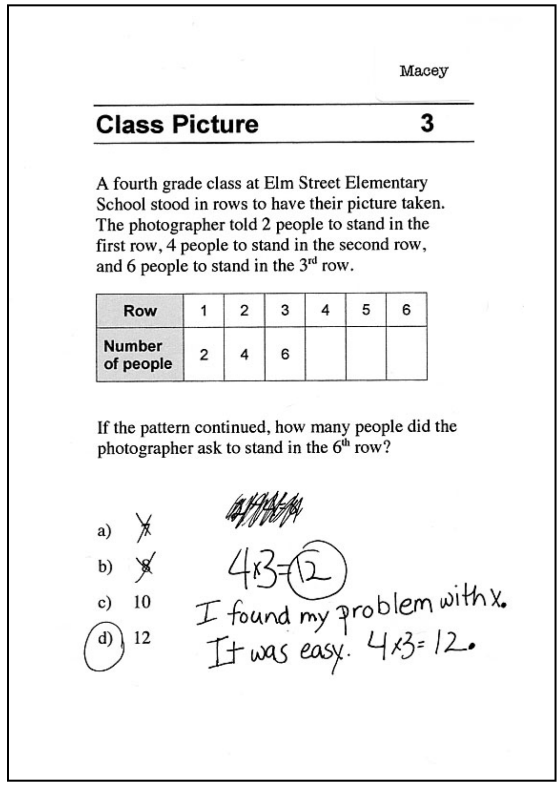
| Row | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of people | 2 | 4 | 6 | 8 | 10 | 12 |

If the pattern continued, how many people did the photographer ask to stand in the 6th row?

a) 7
b) 8
c) 10
d) 12 *

8, 10, 12
2 + 4 +
2 + 4 + 6 = 12
2, 4, 6, 8, 10, 12

*I saw that Macey had circled 12 on her paper but had written 4 x 3 = 12.  That puzzled me. I wondered whether she too thought she needed to write a number sentence to justify her answer.  I asked her to go to the overhead and show us what she was thinking.*


Macey

**Class Picture** **3**

A fourth grade class at Elm Street Elementary School stood in rows to have their picture taken. The photographer told 2 people to stand in the first row, 4 people to stand in the second row, and 6 people to stand in the 3rd row.

| Row | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of people | 2 | 4 | 6 | | | |

If the pattern continued, how many people did the photographer ask to stand in the 6th row?

a) ~~1~~
b) ~~8~~
c) 10
d) 12

4x3=12
I found my problem with x.
It was easy. 4x3=12.

**Macey:** I found my problem with times.  It was easy.  Four times three equals twelve (*as she circles 12 without completing the table*).

**Teacher:** Why did you do four times three equals twelve?

**Macey:** because (*pointing to the empty spaces in the table below 4, 5, 6*) eight, ten, twelve…(*as she writes 8, 10, 12 below 4, 5, 6, then wipes them off and writes 4 x 3=12*)

**Teacher:** How did you know to write four times three?

**Macey:** Because I knew four is closest to twelve, and four times three is twelve

**Teacher:** Why not choose six? Isn't six closer to twelve?

**Macey:** hmmmm…four! Because four times three is twelve!

**Teacher:** But how about six times two?

**Macey:** Also twelve…But I say four.

**Teacher:** But why four?  Why is four better than six?

**Macey:** Four (*pointing 4 in the top row of the table*) TIMES one, two, three (*pointing to each of the three empty spaces in the bottom row of the table and counting them as she pointed*)!

"four times"

| Row | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of People | 2 | 4 | 6 | | | |

"one" "two" "three"

*Macey had come up with the correct answer but her reasoning was faulty.  As she explained I realized she began with 4 – the first number in the table with a blank cell below it – and then multiplied by 3 – the number of blank cells in the table.*

*In both instances, I realized how important it is to discuss how students arrived at a correct answer as well as a wrong answer.  Clearly a right answer did not signal that a student necessarily had a full grasp on the problem.*