

Functional Mathematics

for educated adults

A few thought-provoking tasks
that any well-educated adult could, and should, be able to do
– *without having been taught the specific problem.*

(see commentary on page 4).

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Sudden Infant Deaths = Murder?

In the general population, about 1 baby in 8,000 dies in an unexplained "crib death". The cause or causes are at present unknown. Three babies in one family have died. The mother is on trial. An expert witness says:

"One crib death is a family tragedy; two is deeply suspicious; three is murder. The odds of even two deaths in one family are 64 million to 1"

Discuss the reasoning behind the expert witness' statement, noting any errors, and write an improved version to present to the jury.

Conference budget

Your job is to plan a conference budget, using a computer spreadsheet.

You have already made a start:

| A | B | C | D | E | F |
|-----------------|-------------------------------|---|-----------|----------------|----|
| College charges | | | Delegates | .@ \$ each | \$ |
| Monday | Buffet Supper | | 30 | 17.00 | 0 |
| | Single En-suite Accommodation | | 30 | 40.00 | 0 |
| Tuesday | Breakfast | | 30 | 8.00 | 0 |
| | Morning Coffee | | 30 | 1.90 | 0 |
| | Lunch | | 30 | 15.00 | 0 |
| | Afternoon tea | | 30 | 1.90 | 0 |
| | Dinner served | | 40 | 50.00 | 0 |
| | Single room | | 30 | 40.00 | 0 |
| | Plenary Room | | 30 | 15.77 | 0 |
| | Breakout rooms | | 2 | 85.10 | 0 |
| Wednesday | Breakfast | | 30 | | 0 |
| | Morning Coffee | | 30 | | 0 |
| | Lunch | | 30 | | 0 |
| | Afternoon tea | | 30 | | 0 |
| | No Dinner | | 30 | | 0 |
| | Single room | | 30 | | 0 |
| | Plenary Room | | 30 | | 0 |
| | Breakout rooms | | 2 | | 0 |
| Thursday | Breakfast | | 30 | 8.00 | 0 |
| | Morning Coffee | | 30 | 1.90 | 0 |
| | Lunch | | 30 | 15.00 | 0 |
| | Afternoon tea | | 30 | 1.90 | 0 |
| | Plenary Room | | 30 | 15.77 | 0 |
| | Breakout rooms | | 2 | 85.10 | 0 |
| | | | | Total charges> | 0 |
| | | | | Sales Tax @ 8% | 0 |
| | | | | Total | 0 |

(i) Complete the entries for Wednesday in column D.

(ii) Calculate appropriate totals in column E.

(The spreadsheet was on a computer; here, work out what you would do)

Elementary school teachers

In a country with 300 million people, about how many elementary school teachers will be needed? Try to estimate a sensible answer using your own everyday knowledge about the world. Write an explanation of your answer, stating any assumptions you make.

Bike or Bus

Terry is soon to go to secondary school. There is no school bus. The bus trip costs \$1.50 and Terry's parents are considering the alternative of buying him a bicycle.

Help Terry's parents decide what to do by carefully working out the relative merits of the two alternatives.

Scheduling Traffic Lights

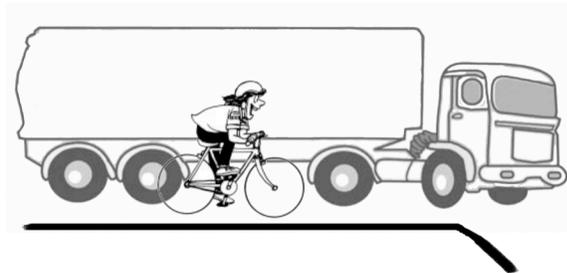
A new set of traffic lights has been installed at an intersection formed by the crossing of two roads. Left turns are NOT permitted at this intersection.

For how long should each road be shown the green light?
Explain your reasoning clearly.

Right turns

The truck is stopped at traffic lights, planning to turn right. The cycle is alongside.

If the cyclist waits for the truck to turn before moving, what will happen?
Explain why this will happen with a diagram.



What would be your advice:

- to the truck driver?
- to the cyclist?

Give reasons in each case.

Being realistic about risk

"My sixty year old mother, who lives in New York, gets frightened by newspapers. One day she is afraid of being a victim of crime, the next she is frightened of being killed in a road accident, then it's terrorists, and so on."

(i) Use a website with appropriate statistics to estimate the chances of my mother being a victim of these events, and others you think she might worry about.

(ii) Write down some reassurance you would give her – and compare the likelihood of these events with her 'base risk' the probability that women of her age will die during the coming year.

Commentary on the tasks, and responses to them

Sudden Infant Deaths = Murder?

What we expect here is not a full statistical analysis, which would need more information, but a recognition that the reasoning presented is deeply flawed. There are two elementary mistakes in the statement, and one that is a bit more subtle. It would be correct to say:

1. The chance of these deaths being entirely *unconnected* chance events is very small indeed – if there has been one death, the chance of two more unconnected deaths is about 64 million to one.
2. What can the connection be? It may be that the mother killed the children; on the other hand, particularly since we do not understand the cause(s) of crib death, there may be other explanations. For many conditions (cancer and heart disease, for example) genetic and environmental factors are known to affect the probability substantially.

Any lawyer or judge with functional mathematics should have seen problems with the witness statement. It is not lack of basic skills that was their failing (They could surely have worked out the chance of a double six on rolling two dice as $1/36$) but an understanding of the necessary assumptions. (The woman in similar cases are only now being released)

Conference budget

This is a task we give (on a working spreadsheet) to candidates for the post of Secretary/Administrator in the team. Most are graduates. All "know Excel". None complete the task. Most see that Wednesday's values in Column D are probably the same as Tuesday's and Thursday's. Few enter the appropriate, or indeed any, formulas in Column E (Formulating relationships is a basic piece of algebra that is neglected in schools – and maths tests). Some even work out the row totals on a calculator, entering the *values!*

Elementary school teachers

This kind of back-of-the-envelope calculation is an important life skill. Here it requires choosing appropriate facts (6 years in elementary school out of a life of 60-80 years, one teacher for 20-30 kids), and formulating appropriate proportional relationships giving $(300*6)/(70*25) \sim 1$ million primary teachers (to an accuracy appropriate to that of the data) This kind of linkage with the real world, common in the English Language Arts curriculum, is rare in school Mathematics (and absent in tests).

Bike or Bus and Scheduling Traffic Lights as for *Ice Cream Van* on the other test.

Right turns Functional mathematics often involves space and shape, too.

Being realistic about risk

Education, and functional mathematics in particular, can help narrow the gap between perceived and real risk. Given the power of anecdote over evidence, exploited daily by the media, this is a major challenge; meeting it could make a huge contribution to people's quality of life, and that of their children. Few people have any sense of the magnitude of specific risks, or any idea of the unavoidable 'base risk' for someone of their age. (Note that only order-of-magnitude estimates, not accurate numbers, are relevant here)

Explicitly teaching students to use their mathematics on real problems is now proven, with typical teachers; it is essential to functionality. These exemplars also show how deterministic and statistical reasoning intermesh in functional mathematics.