Functional Mathematics in High School

A few thought-provoking tasks that any well-educated student should be able to do by age 16
- *without having been taught the specific problem*

*(see commentary on page 4).*

from
MARS: Mathematics Assessment Resource Service
Shell Centre for Mathematical Education
University of Nottingham

Summer 2005
At the airport

<table>
<thead>
<tr>
<th>Currency</th>
<th>We Buy</th>
<th>We Sell</th>
</tr>
</thead>
<tbody>
<tr>
<td>£ UK Pound</td>
<td>$ 1.69</td>
<td>$ 1.87</td>
</tr>
<tr>
<td>€ Euro</td>
<td>$ 1.112</td>
<td>$ 1.237</td>
</tr>
</tbody>
</table>

No commission!

(a) How many Euros (€) would you get for $500 (Dollars)?
(b) How many Dollars ($) can you get for £700 (Pounds)?
(c) How much would you have to pay, in Dollars and Cents, to get exactly £550?

Freeway journey

I think we need to stop for gas before we reach L.A.

No, we'll be OK.
The tank holds about 15 gallons, and I filled it up yesterday.
We haven't got time to stop.

How many miles does this car get to a gallon?

On the freeway, at this speed, about 35 miles per gallon.

(i) Do they have to stop for gas? Explain your reasoning.
(ii) Suppose they decide to stop for 10 minutes.
At what time will they reach Los Angeles?
Ice cream van
You are considering driving an ice cream van during the Summer break. Your friend, who “knows everything”, says that “It’s easy money”. You make a few enquiries and find that the van costs $600 per week to hire. Typical selling data is that one can sell an average of 30 ice creams per hour, each costing 50c to make and each selling for $1.50.

How hard will you have to work in order to make this “easy money”? Explain your reasoning clearly.

Paper clips
This paper clip is just over 4 cm long.

How many paper clips like this may be made from a straight piece of wire 10 metres long?

Cold calling
The following is part of a genuine letter of complaint to a bank.

“I would like to complain about the behaviour of XYZ Bank and the advice given during a recent unsolicited telephone call. Having been told I was "pre-approved" for a $5,000 loan, the operator asked me for my financial details. I told her that I currently had two credit cards, one with a balance of $3000 and one with $1000. She said that they could consolidate these debts into a single payment which would be cheaper. I pressed her on the APR which she explained was 16.4%, which caused me to decline the loan because my two credit cards are currently at 7% and 9.9% APR respectively. The operator then informed me that their loan would work out cheaper, because 7% and 9.9% works out at 16.9%, nearly 0.5% higher than the bank loan.”

(i) Explain what is wrong with the operator’s reasoning.
(ii) How much more expensive is the bank’s consolidated loan?
**Commentary on the tasks:**

**At the airport**  
It is interesting to compare this with a question from a current school test:

The table shows the exchange rates between different currencies:

<table>
<thead>
<tr>
<th>Currency</th>
<th>Conversion Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>£1 (Pound)</td>
<td>€1.45 (Euros)</td>
</tr>
<tr>
<td>$1 (Dollar)</td>
<td>€0.81 (Euros)</td>
</tr>
</tbody>
</table>

(a) Jane changes £400 into euros. How many euros does she receive?  
(b) Sonia changes £672 euros into dollars. How many dollars does she receive?

Note how the simplification of the presentation leaves a major gap from real functionality. This unreality, characteristic of secondary school mathematics, confirms many students' view that the subject has no relevance to their lives.

**Freeway journey**  
From an actual test. Most examples of functional mathematics have been eliminated in the fragmentation of tasks to assess separate micro-skills.

**Ice cream van**  
This task was used in a research study of the performance of 120 very able 17 year old students. Many solved the tasks, using arithmetic and, sometimes graphs. *None used algebra*, the natural language for formulating such problems. Their algebra was non-functional, despite 5+ years of high success in the standard imitative inward-looking algebra curriculum.

**Paper clips** exemplifies a step towards functionality; a school math version is:

(b) A semi-circle has a diameter of 12 cm. Calculate the perimeter.

**Cold calling** – a common misconception, and con, to unravel.

*Explicitly teaching students to use their mathematics on real problems is now proven, with typical teachers; it is essential to functionality. These exemplars also show how deterministic and statistical reasoning intermesh in functional mathematics.*